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COLMATATION-SUFFOSION FILTRATION IN A POROUS MEDIUM WITH MOBILE AND IMMOBILE FLUIDS

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The problem of colmatation-suffosion filtration of disperse systems in a two-dimensional porous medium consisting of two zones with mobile and immobile fluids has been studied with account for diffusion phenomena. The effects of lateral and longitudinal diffusion and colmatation-suffosion phenomena on collector properties of a porous medium have been determined.

In the past years the problems of transfer of inhomogeneous fluids in porous media have become of ever greater significance in connection with the extensive use of tertiary methods of acting on oil strata, the wide introduction of the ways of utilizing various domestic and industrial wastes, the investigation of filtration-capacitance properties of subsurface reservoirs through injecting fluids with various additives serving as tracer substances, etc. [1–5]. In view of this, the problems of the mathematical modeling of substance transfer in porous and cracked porous media are being actively discussed. A few works, which are devoted to this topic, can be named [6–8]. Approaches to the modeling are based on the law of balance of the substance in a certain check volume with the use of additional phenomenological relations. Problems arising in the study of transfer of substances dissolved in stratum fluids can be divided into two groups: 1) substance transfer without regard to the change in filtration properties of the medium, and 2) a change in the structure of the pore volume during transfer due to settling of a part of a dissolved (or spherical) substance, which necessarily affects filtration properties of the medium. While approaches to the modeling are similar, in the second case it is essential to take into account the dependence of collector properties of the porous medium on the degree and character of filling of the pore volume.

Multidimensional problems in which hydrodynamic dispersion in various directions of the space proceeds with a dissimilar intensity have been studied little. Furthermore, the problems of multidimensional colmatation-suffosion filtration where colmatation-suffosion phenomena are considered along with longitudinal and lateral diffusion have not been investigated until now. As is shown in [9, 10], the presence of zones with poor penetrability in macroscopically inhomogeneous media has a marked effect on the processes of fluid motion and substance transfer in them. As a rule, in these zones a fluid is slightly mobile or immobile, which makes it possible to treat them as stagnant. In the filtration of a fluid with suspended particles in such media, motion generally occurs in highly penetrable zones. However, the presence of zones with an immobile fluid has a noticeable effect on substance transfer, since the substance is transferred by diffusion from zones with a mobile fluid to zones with an immobile one and vice versa. It should be noted that problems of the modeling of substance transfer in the medium with zones of mobile and immobile (so-called "dead" zones) fluids were first studied in [11–13].

Using methodologies [14–17], we consider the problem of colmatation-suffosion filtration in a porous medium with two zones: 1) with a mobile fluid and 2) with an immobile fluid. Let the region of investigation of the problem consist of two parts, viz., $R^+\{0 \le x < \infty,\ 0 \le y \le h\}$ and $R^-\{0 \le x < \infty,\ -\infty < y \le 0\}$ (Fig. 1). Initially, the regions R^+ and R^- are filled with a fluid without particles. Starting with $t > 0$, a fluid with the concentration of solid particles c_0 = const enters the region R^+ through the boundary $x = 0$, $0 \le y \le h$. A fluid moves in R^+ with a constant averaged velocity u. The averaging is made over the void part of the cross section of the zone R^+.

Diffusive transfer of particles proceeds by the classical Fick's law

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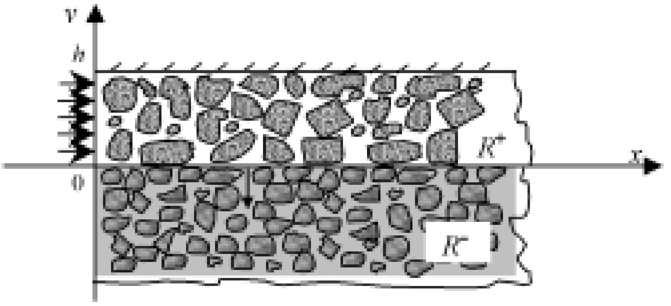

Fig. 1. Schematic of colmatation-diffusion substance transfer in a two-dimensional region.

$$\mathbf{q} = -\,\mathbf{D}\,\nabla c\,, \tag{1}$$

where in the general case the diffusion tensor **D** for plane transfer of the substance is of the form

$$D = \begin{pmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{pmatrix}.$$

Let the direction of substance motion coincide with one of the principal axes (for example, x), which gives $D_{11} = D_1$, $D_{12} = D_{21} = 0$, and $D_{22} = D_2$, i.e.,

$$D = \begin{pmatrix} D_1 & 0 \\ 0 & D_2 \end{pmatrix}.$$

Then, the longitudinal and lateral substance fluxes are expressed as

$$q_x = -\,D_1 \frac{\partial c}{\partial x}\,, \quad q_y = -\,D_2 \frac{\partial c}{\partial y}\,. \tag{2}$$

The upper boundary of the region R^+ is impenetrable to a fluid and particles. A fluid moves only in the direction of x in the region R^+. Here, the passing of suspended particles deeper into the region is accompanied by their settling (colmatation) followed by a partial breakaway from the trapped (settled) state and a subsequent transfer to other pores (suffosion). We assume that the kinetics of the colmatation and suffosion process is described by the equation previously proposed in [14]. Under these assumptions, with allowance for Eqs. (1) and (2) colmatation-suffosion transfer in the region R^+ can be described by the equations

$$\frac{\partial c}{\partial t} + u \frac{\partial c}{\partial x} = D_1 \frac{\partial^2 c}{\partial x^2} + D_2 \frac{\partial^2 c}{\partial y^2} + \frac{1}{\varepsilon_0} \frac{\partial \varepsilon}{\partial t}\,, \tag{3}$$

$$\frac{\partial \varepsilon}{\partial t} = \omega_1 \left(\varepsilon_0 - \varepsilon\right) \left|\nabla p\right| - \omega_2 \varepsilon c\,. \tag{4}$$

In the region R^- only diffusive substance transfer occurs. Taking into account its poor penetrability we assume that substance transfer proceeds with a certain effective diffusion coefficient D_3 different from the molecular diffusion coefficient D_0. Furthermore, in this region the lateral diffusion coefficient characterizing substance transfer in the direction of x is disregarded. Then, substance transfer in R^- can be described by the conventional diffusion equation

$$\frac{\partial c}{\partial t} = D_3 \frac{\partial^2 c}{\partial y^2}\,, \quad (x, y) \cup R^-\,. \tag{5}$$

The field of filtration rate $w = u\varepsilon$ is defined by the Darcy law

$$\mathbf{w} = -K(\varepsilon)\,\nabla p\,. \tag{6}$$

The filtration coefficient $K(\varepsilon)$ can be specified in the form of a linear or nonlinear function of porosity [14–17]. In the linear case it is assumed that $K(\varepsilon) = k_0\varepsilon$, k = const, and from among the nonlinear dependences the most commonly used is the von Kármán–Kozeny law $K(\varepsilon) = k_0\varepsilon^3/(1-\varepsilon)^2$. It should be noted that, in the linear case of the variation in $K(\varepsilon)$, from expression (6) we obtain the field of pressure gradient $|\nabla p| = u/k_0$, and when using the von Kármán–Kozeny law we have

$$|\nabla p| = \frac{u(1-\varepsilon)^2}{k_0\varepsilon^2}\,. \tag{7}$$

Through the common boundary of the regions R^+ and R^-, the mass of the substance (particles) is transferred to R^+ when $x \to \infty$ and to R^- when $y \to -\infty$. Presumably, the substance flux is absent. This, in fact, means that such initial stage of the additive propagation over the stratum is considered, at which concentration profiles do not yet reach the boundary $x \to \infty$ in R^+ and $y \to -\infty$ in R^-. On the common boundary of the regions R^+ and R^-, i.e., on $y = 0$, we require that the variation in the substance concentration and substance flux be continuous. With the formulated constraints, the initial and boundary conditions of the problems are of the form

$$c = 0 \quad \text{in} \quad R^+ \cup R^- \quad \text{when} \quad t = 0\,, \tag{8}$$

$$\varepsilon = \varepsilon_0 \quad \text{in} \quad R^+ \quad \text{when} \quad t = 0\,, \tag{9}$$

$$c = c_0\,, \;\; x = 0\,, \;\; 0 \le y \le h\,, \tag{10}$$

$$\frac{\partial c}{\partial x} = 0\,, \;\; x = \infty\,, \;\; 0 \le y \le h\,, \tag{11}$$

$$\frac{\partial c}{\partial y} = 0\,, \;\; y = h\,, \;\; 0 \le x < \infty\,, \tag{12}$$

$$c\,\big|_{y=+0} = c\,\big|_{y=-0}\,, \tag{13}$$

$$D_2 \frac{\partial c}{\partial y}\bigg|_{y=+0} = D_3 \frac{\partial c}{\partial y}\bigg|_{y=-0}\,, \tag{14}$$

$$\frac{\partial c}{\partial y} = 0\,, \;\; y = -\infty\,, \;\; 0 \le x < \infty\,. \tag{15}$$

Problem (3)–(15) is solved using the finite-difference method. Although Eqs. (3) and (5) are linear, even with the linearization of Eq. (4) the solution of the problem by one of analytical methods involves some difficulties and, as a rule, the use of analytical methods leads to cumbersome calculations, which renders the determination of characteristics not an easy procedure. Let us construct a grid in the region $R^+ \cup R^-$: $\omega_{h_1h_2\tau} = \omega^+_{h_1h_2\tau} \cup \omega^-_{h_1h_2\tau}$, where

$$\overset{+}{\omega}_{h_1h_2\tau} = \Big\{(t_k, x_i\,,\ y_j)\,,\ t_k = \tau k\,,\ x_i = ih_1\,,\ y_j = j\overset{+}{h}_2\,,\ k = \overline{0, K}\,,$$

$$i = 0, 1, \ldots,\ j = 0, 1, \ldots, J^+,\quad \tau = T/K\,,\ \overset{+}{h}_2 = h/J^+\Big\}\,,$$

$$\overset{-}{\omega}_{h_1h_2\tau} = \Big\{(t_k, x_i\,,\ y_j)\,,\ t_k = \tau k\,,\ x_i = ih_1\,,\ y_j = -\overset{-}{h}_2 j\,,\ k = \overline{0, K}\,,\ i = 0, 1, \ldots,\ j = 0, 1, \ldots,\ \tau = T/K\Big\}\,.$$

On this grid, Eqs. (3) and (5) are approximated as follows:

$$\frac{\overset{k+1/2}{c}_{ij} - \overset{k}{c}_{ij}}{0.5\tau} + u\,\frac{\overset{k+1/2}{c}_{ij} - \overset{k+1/2}{c}_{i-1,j}}{h_1} = D_1\Lambda_1\overset{k+1/2}{c}_{ij} + D_2\overset{+}{\Lambda}_2\overset{k}{c}_{ij} + \frac{1}{\varepsilon_0}\,\frac{\overset{k+1/2}{\varepsilon}_{ij} - \overset{k}{\varepsilon}_{ij}}{0.5\tau}\,,$$

$$i = \overline{1, I-1}\,,\quad j = \overline{1, J^+ - 1}\,,\quad k = 0, 1, \ldots, \tag{16}$$

$$\frac{\overset{k+1}{c}_{ij} - \overset{k+1/2}{c}_{ij}}{0.5\tau} + u\,\frac{\overset{k+1}{c}_{ij} - \overset{k+1}{c}_{i-1,j}}{h_1} = D_1\Lambda_1\overset{k+1/2}{c}_{ij} + D_2\overset{+}{\Lambda}_2\overset{k+1}{c}_{ij} + \frac{1}{\varepsilon_0}\,\frac{\overset{k+1}{\varepsilon}_{ij} - \overset{k+1/2}{\varepsilon}_{ij}}{0.5\tau}\,,$$

$$i = \overline{1, I-1}\,,\quad j = \overline{1, J^+ - 1}\,,\quad k = 0, 1, \ldots, \tag{17}$$

$$\frac{\overset{k+1/2}{c}_{ij} - \overset{k}{c}_{ij}}{0.5\tau} = D_3\overset{-}{\Lambda}_2\overset{k+1/2}{c}_{ij}\,,\quad i = \overline{0, I}\,,\ \ j = \overline{1, J^- - 1}\,,\ \ k = \overline{0, K-1}\,, \tag{18}$$

$$\frac{\overset{k+1}{c}_{ij} - \overset{k+1/2}{c}_{ij}}{0.5\tau} = D_3\overset{-}{\Lambda}_2\overset{k+1}{c}_{ij}\,,\quad i = \overline{0, I}\,,\ \ j = \overline{1, J^- - 1}\,,\ \ k = \overline{0, K-1}\,, \tag{19}$$

where $\Lambda_1\overset{k}{c}_{ij} = (\overset{k}{c}_{i-1,j} - 2\overset{k}{c}_{ij} + \overset{k}{c}_{i+1,j})/h_1^2$, $\overset{+}{\Lambda}_2\overset{k}{c}_{ij} = (\overset{k}{c}_{i,j-1} - 2\overset{k}{c}_{ij} + \overset{k}{c}_{i,j+1})/(\overset{+}{h}_2)^2$, $\overset{-}{\Lambda}_2\overset{k}{c}_{ij} = (\overset{k}{c}_{i,j-1} - 2\overset{k}{c}_{ij} + \overset{k}{c}_{i,j+1})/(\overset{-}{h}_2)^2$, $\overset{k+1/2}{c}_{ij}$ are values of the grid function on a half time layer $t_{k+1/2} = t_k + 0.5\tau$.

Equation (4) in $\overset{+}{\omega}_{h_1h_2\tau}$ is approximated as follows:

$$\frac{\overset{k+1/2}{\varepsilon}_{ij} - \overset{k}{\varepsilon}_{ij}}{0.5\tau} = \omega_1\Big(\varepsilon_0 - \overset{k}{\varepsilon}_{ij}\Big)\,|\nabla p\,|_{ij}^{k} - \omega_2\overset{k}{\varepsilon}_{ij}\overset{k}{c}_{ij}\,,\quad i = \overline{0, I}\,,\ \ j = \overline{1, J^+}\,,\ \ k = \overline{0, K-1}\,, \tag{20}$$

$$\frac{\overset{k+1}{\varepsilon}_{ij} - \overset{k+1/2}{\varepsilon}_{ij}}{0.5\tau} = \omega_1\Big(\varepsilon_0 - \overset{k+1/2}{\varepsilon}_{ij}\Big)\,|\nabla p\,|_{ij}^{k+1/2} - \omega_2\overset{k+1/2}{\varepsilon}_{ij}\overset{k+1/2}{c}_{ij}\,,\quad i = \overline{0, I}\,,\ \ j = \overline{1, J^+}\,,\ \ k = \overline{0, K-1}\,. \tag{21}$$

The initial and boundary conditions are approximated as

$$\overset{0}{c}_{ij} = 0 \quad \text{in} \quad \omega_{h_1h_2\tau}\,, \tag{22}$$

$$\overset{0}{\varepsilon}_{ij} = \varepsilon_0 \quad \text{in} \quad \overset{+}{\omega}_{h_1h_2\tau}\,, \tag{23}$$

$$\overset{k}{c}_{0j} = c_0\,,\quad j = \overline{0, J^+}\,,\quad k = \overline{0, K}\,, \tag{24}$$

$$c_{I,j}^{k}=c_{I-1,j}^{k}\,,\quad j=\overline{0,J^{+}}\,,\quad k=\overline{0,K}\,, \tag{25}$$

$$c_{i,J^{+}-1}^{k}=c_{i,J^{+}}^{k}\,,\quad i=0,\,1,\,...,\quad k=\overline{0,K}\,, \tag{26}$$

$$c_{i,+0}^{k}=c_{i,-0}^{k}\,,\qquad i=0,\,1,\,...,\quad k=\overline{0,K}\,, \tag{27}$$

$$D_2\frac{c_{i,1}^{k}-c_{i,0}^{k}}{h_2^{+}}=D_3\frac{c_{i,0}^{k}-c_{i,-1}^{k}}{h_2^{-}}\,,\quad i=0,\,1,\,...,\quad k=\overline{0,K}\,, \tag{28}$$

$$c_{i,J^{-}}^{k}=c_{i,J^{-}-1}^{k}\,,\quad i=0,\,1,\,...,\quad k=\overline{0,K}\,, \tag{29}$$

where I and J^{-} are taken fairly large, such that the concentration profiles do not reach the chosen conditional boundaries of the regions.

Equations (16) and (17) are brought to the form

$$A_1c_{i-1,j}^{k+1/2}-B_1c_{ij}^{k+1/2}+E_1c_{i+1,j}^{k+1/2}=-F_{ij}^{(1)}\,,\quad i=\overline{1,I-1}\,,\quad J=\overline{1,J^{+}-1}\,,\quad k=\overline{0,K-1}\,, \tag{30}$$

$$A_2c_{i,j-1}^{k+1}-B_2c_{ij}^{k+1}+E_2c_{i,j+1}^{k+1}=-F_{ij}^{(2)}\,,\quad i=\overline{1,I-1}\,,\quad J=\overline{1,J^{+}-1}\,,\quad k=\overline{0,K-1}\,, \tag{31}$$

where

$$A_1=0.5\tau\left(\frac{D_1}{h_1^2}+\frac{u}{h_1}\right);\quad B_1=1+0.5\tau\left(\frac{2D_1}{h_1^2}+\frac{u}{h_1}\right);\quad E_1=0.5\tau\frac{D_1}{h_1^2}\,;$$

$$F_{ij}^{(1)}=c_{ij}^{k}+0.5\tau\frac{D_2}{\left(h_2^{+}\right)^2}\left(c_{i,j-1}^{k}-2c_{i,j}^{k}+c_{i,j+1}^{k}\right)+\frac{\varepsilon_{ij}^{k+1/2}-\varepsilon_{ij}^{k}}{\varepsilon_0}\,;$$

$$A_2=0.5\tau\left(\frac{D_2}{\left(h_2^{+}\right)^2}+\frac{u}{h_1}\right);\quad B_2=1+0.5\tau\left(\frac{2D_1}{\left(h_2^{+}\right)^2}+\frac{u}{h_1}\right);\quad E_2=0.5\tau\frac{D_2}{\left(h_2^{+}\right)^2}\,;$$

$$F_{ij}^{(2)}=c_{ij}^{k+1/2}+0.5\tau\frac{D_1}{h_1^2}\left(c_{i-1,j}^{k+1/2}-2c_{i,j}^{k+1/2}+c_{i+1,j}^{k+1/2}\right)+\frac{\varepsilon_{ij}^{k+1}-\varepsilon_{ij}^{k+1/2}}{\varepsilon_0}\,.$$

Equations (18) and (19) are brought to the form similar to Eqs. (30) and (31):

$$A_3c_{i,j-1}^{k+1/2}-B_3c_{ij}^{k+1/2}+E_3c_{i,j+1}^{k+1/2}=-F_{ij}^{(3)}\,, \tag{32}$$

$$A_3c_{i,j-1}^{k+1}-B_3c_{ij}^{k+1}+E_3c_{i,j+1}^{k+1}=-F_{ij}^{(4)}\,, \tag{33}$$

where

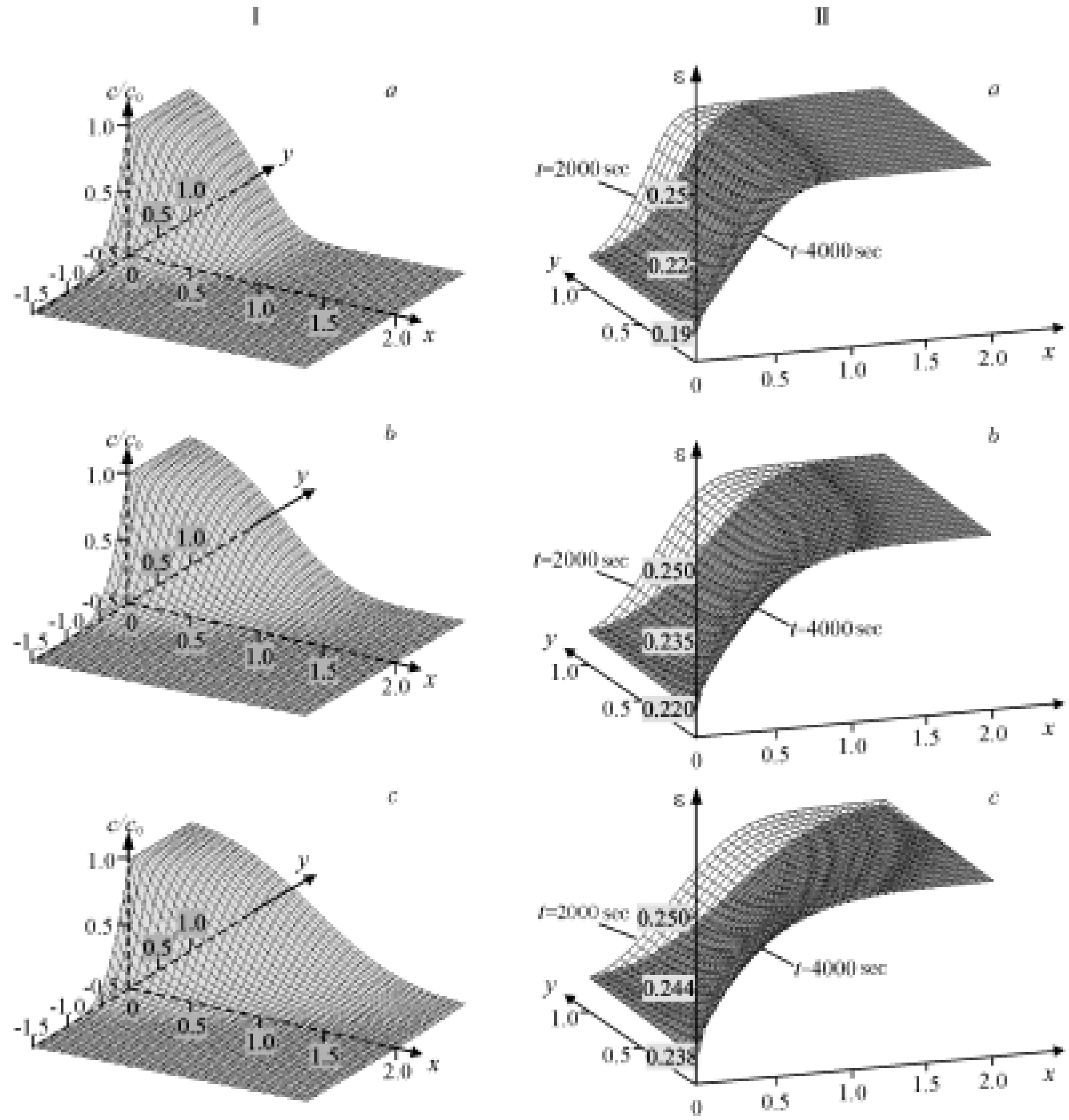

Fig. 2. Surfaces of the relative concentration (I) and porosity (II) in R^+ at $D_1 = 5 \cdot 10^{-5}$ m^2/sec, $D_2 = 10^{-5}$ m^2/sec, and $D_3 = 10^{-5}$ m^2/sec: a) $\omega_1 = 0.1$ m/(MPa·sec) and $\omega_2 = 0.1$ sec^{-1}, b) $\omega_1 = 0.1$ m/(MPa·sec) and $\omega_2 = 0.1$ sec^{-1}, and c) $\omega_1 = 0.3$ m/(MPa·sec) and $\omega_2 = 0.3$ sec^{-1}.

$$A_3 = 0.5\tau \frac{D_3}{\left(h_2^-\right)^2}\,; \quad B_3 = 1 + \tau \frac{D_3}{\left(h_2^-\right)^2}\,; \quad E_3 = 0.5\tau \frac{D_3}{\left(h_2^-\right)^2}\,; \quad F_{ij}^{(3)} = c_{ij}^{k}\,; \quad F_{ij}^{(4)} = c_{ij}^{k+1/2}\,.$$

Equations (30) and (31) are determined in $\omega^+_{h_1h_2\tau}$, and Eqs. (32) and (33), in $\omega^-_{h_1h_2\tau}$. From Eq. (7) it is possible to obtain the field of pressure gradient

$$|\nabla p|_{ij}^{k+1} = \frac{u\left(1 - \varepsilon_{ij}^{k+1}\right)^2}{k_0\left(\varepsilon_{ij}^{k+1}\right)^2}\,, \quad i = \overline{0, I}\,, \quad j = \overline{0, J^+}\,, \quad k = \overline{0, K}\,. \tag{34}$$

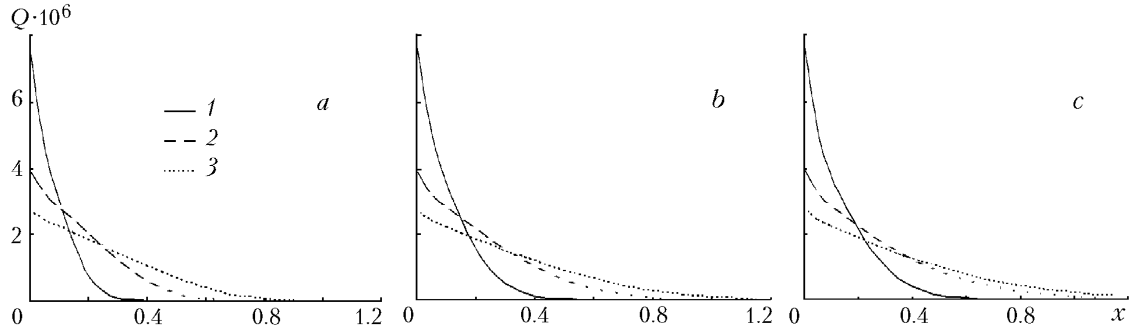

Fig. 3. Variation in the relative rate of flow of the substance Q on the boundary of regions R^{+} and R^{-} at various instants of time t: 1) 500, 2) 2000, and 3) 4000 sec. D_1–D_3; a–c, designations same as in Fig. 2.

The following order the calculation is set. Calculating $|\nabla p|_{ij}^{k}$ from Eq. (34) (using k in lieu of $k+1$), we determine $\varepsilon_{ij}^{k+1/2}$ from Eq. (20). Thereafter we solve Eqs. (30) and (32) and find $c_{ij}^{k+1/2}$. Having determined the field of pressure gradient $|\nabla p|_{ij}^{k+1/2}$ according to Eq. (34) (using $k+1/2$ in lieu of $k+1$), with the aid of these values in Eq. (21) we obtain $\varepsilon_{ij}^{k+1/2}$ in $\omega^{+}_{h_1h_2\tau}$. Solving Eqs. (31) and (33) we find $c_{ij}^{k+1/2}$ in $\omega_{h_1h_2\tau}$.

Equations (30) and (31) are solved by the method of marching in the directions of i and j, respectively. Equations (32) and (33) are also solved using the method of marching, but only in the direction of the variation in j. In the calculations we used the following values of the initial parameters: $u = 6 \cdot 10^{-4}$ m/sec, $k_0 = 10^{-2}$ m^2/(MPa·sec), $h = 1$ m, $c_0 = 0.01$, and $\varepsilon = 0.25$, and various values of D_1, D_2, D_3, ω_1, and ω_2.

Some calculated results are presented in Fig. 2. Figure 2, I shows surfaces of the relative concentrations c/c_0 at 4000 sec and various ω_1 and ω_2. An increase in the substance concentration on the surfaces in the region $R^{+} \cup R^{-}$ can be observed, with substance transfer to R^{-} becoming significant at small x. Deformation of the concentration surfaces due to substance transfer from R^{+} to R^{-} does not reach the boundary $y = h$ in R^{+} (Fig. 2, I, a). However, a broadening of the zone with decrease in c/c_0 in R^{+} with time toward large y is noticeable. Evidently, at large t, the front of decrease in c/c_0 due to diffusive substance transfer from R^{+} to R^{-} can be expected to reach the boundary $y = h$ in R^{+}. Due to diffusive substance transfer from R^{+} to R^{-} at small y in R^{+} the concentration surfaces diminish in comparison with such for relatively large y. As is seen from Fig. 2, I, a and b, the weakening of colmatation and the augmentation of suffosion have a marked effect on the distribution of the substance concentration in R^{+} and therefore, in R^{-}. In Fig. 2, I, b and c, one can observe an appreciable advance of the concentration surfaces toward large x. At small $y \cup R^{+}$ the concentrations are substantially higher than in the case with Fig. 2, I, a. As a result, the concentration surfaces in R^{-} intrude much deeper and wider. From the physical standpoint this is because, with weak colmatation and pronounced suffosion, relatively few particles settle in pores; therefore, a major portion of particles remains in suspension in a moving fluid. This affects the extent to which the pore volume is filled with particles. Within the framework of the proposed model, the particle settling results in a closing of the fluid and particle passages in the pore volume. Therefore, the colmatation and suffosion phenomena directly influence the active porosity. The porosity dynamics in the above three regimes of colmatation and suffosion is shown in Fig. 2, II. Comparing the presented results, one can observe a decrease in the porosity in the colmatation and suffosion zone in R^{+}. Here, due to diffusive substance transfer from R^{+} to R^{-} at small $y \cup R^{+}$, the decrease in ε is not as noticeable as at relatively large y. A delay in the decrease in ε is observed at certain x starting with $x = 0$ where the rate of diffusive substance transfer from R^{+} to R^{-} is appreciable. At fairly large x, to which the zone of the concentration variation did not extend, and therefore, where the effects of diffusive substance transfer from R^{+} to R^{-} were insignificant, such dynamics of ε is not the case. Proceeding from this, a conclusion can be drawn that diffusive substance transfer from R^{+} to R^{-} has a significant influence on the colmatation-suffosion phenomena in R^{+}.

The relative rate of flow of the substance through the boundary $y = 0$ is defined as

$$Q = D_3 \left|\frac{\partial c}{\partial y}\right|\Bigg|_{y=0} ,$$

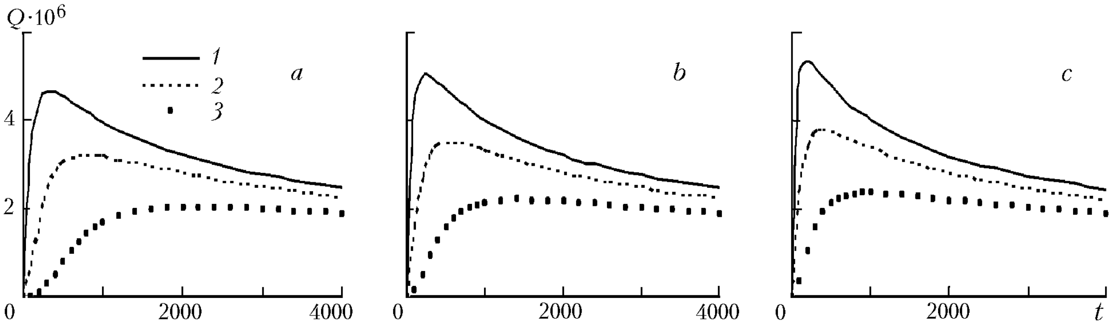

Fig. 4. Variation in Q with t at various x: 1) 0.05, 2) 0.10, and 3) 0.20 m. D_1–D_3; a–c, designations same as in Fig. 2.

which after approximation gives

$$Q = D_3 \left| \frac{c_{i0}^{k+1} - c_{i,-1}^{k+1}}{h_2^-} \right| .$$

For convenience, here we keep the same notation for Q as for its grid values.

Results of calculations of Q are presented in Fig. 3. Analysis of the graphs shows that the weakening of colmatation and the augmentation of suffosion lead to an advance of the zone of diffusive mass transfer through $y = 0$ over relatively large distances along x. At small x, after a sharp increase the values of Q decrease with t and increase at large x. Such dynamics of Q at fixed points is shown in Fig. 4. It is seen that here Q reaches a local maximum, it being more pronounced at the points close to $x = 0$. With increasing suffosion and decreasing colmatation the values of the maximum of Q become larger, with the maxima per se attained at relatively smaller times. This is explained by more intense dynamics of the substance concentration in R^+ and an increase in the rate of mass transfer through $y = 0$.

The obtained results indicate that the presence of the zone with an immobile fluid has a marked influence on substance transfer in the medium with a mobile fluid. The character of mass transfer between the zones with mobile and immobile fluids is determined, among other factors, by the colmatation-suffosion effects in the zone with a mobile fluid.

As a further development of the work, we may indicate the possibility of allowing for the limiting suffosion pressure gradient [18] in the zone with a mobile fluid, which can lead to a "breaking" of the concentration surfaces and relative mass transfer through the common boundary of the media with mobile and immobile fluids.

NOTATION

c_0 and c, constant and running volumetric concentrations of the substance, $\mathrm{m}^3/\mathrm{m}^3$; D_0, molecular diffusion coefficient, m^2/sec; D_1 and D_2, coefficients of longitudinal and lateral convective diffusion in R^+, m^2/sec; D_3, diffusion coefficient in R^-, m^2/sec; h, thickness of the zone with a mobile fluid, m; h_1, grid spacing in the direction of x, m; h_2^+, grid spacing in the direction of y in R^+, m; h_2^-, grid spacing in the direction of y in R^-, m; I, number of the grid intervals along x; J^+, number of the grid intervals along y in R^+; J^-, number of the grid intervals along y in R^-; K, number of the grid intervals with respect to t; $K(\varepsilon)$, filtration coefficient, m^2/(MPa·sec); p, pressure, MPa; $|\nabla p|$, modulus of the pressure gradient p, MPa/m; $\mathbf{q}$, substance flux per unit area, m/sec; Q, relative rate of flow of the substance through the boundary of the media with mobile and immobile fluids, m/sec; q_x and q_y, longitudinal and lateral substance fluxes per unit area, m/sec; R^+ and R^-, zones with mobile and immobile fluids; t, time, sec; T, maximum time during which the process is studied, sec; u, averaged fluid velocity in R^+, m/sec; w, running filtration rate in R^+, m/sec; x and y, coordinates, m; ε_0 and ε, initial and running porosity in R^+; ω_1 and ω_2, coefficients characterizing the intensity of suffosion and colmatation of pores, respectively, m/(MPa·sec), sec^{-1}; τ, time step of the grid, sec.

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